frictional model, the Voellmy model typically produces better simulations of velocity and deposit distribution.

2.1.3 The Bingham rheology

The Bingham resistance model combines plastic and viscous behaviour. The shear resistance is determined by solving the following cubic equation:

$$\tau_{zx}^3 + 3\,(\tau_{yeild}/2 + \mu_{Bingham}\, v_x h)\,\tau_{zx}^2 - \tau_{yeild}^3/2 = 0 \qquad [3]$$

where τ_{yeild} is the Bingham yield stress and $\mu_{Bingham}$ is the Bingham viscosity. The Bingham model may produce better simulations of events involving clayey or highly plasticity materials.

2.2 Calibration

For all case histories, the runout path is based on detailed topographic maps. The material rheologies are determined through empirical calibration. Systematic trial and error back-analyses of similar previous events are used to constrain the rheological relationship and the specific resistance parameter values. Validity of the model is assessed through comparison of calculated and observed landslide characteristics, such as deposit distribution and extent, velocity profiles, and volume. In the December 2007 benchmarking exercises at the International Landslide Risk Management Forum in Hong Kong, the DAN-W and DAN3D models produced results consistent with those of other models from several countries around the world.

3. CASE STUDIES

The following summarizes back-analyses of a number of notable Canadian landslide case histories. The purpose is to show patterns that are gradually emerging in the selection of rheologies and parameters for a variety of landslide types. A more thorough calibration of the models against a large number of case histories and a generalization of the results into a consistent framework of modelling methodology is presently being conducted by the first author.

3.1 Avalanche Lake, Northwest Territories

The prehistoric Avalanche Lake rockslide in the Mackenzie Mountains was exceptionally mobile. Approximately 200 x 10^6 m^3 of dolomite detached as a slab, entraining saturated sediments from the valley floor. The mass disintegrated during the 1220 m decent from the source slope, crossed a wide glacial valley floor and rose up onto a shelf 640 m above the valley floor. Only the leading edge of the slide mass reached the shelf; the bulk of the material returned back into the valley and ran back up the original source slope before settling in the valley. A second lobe flowed for 4 km along a tributary valley (Evans 1989).

Analysis of this event using fluid dynamics requires rigidity in the body of the flow. This may be because the bulk of the debris is dry and angular with substantial internal strength. Using the DAN-W model with the Voellmy rheology with f = 0.02 and ξ = 250 m/s², runup of the leading edge onto the shelf was successfully simulated. Furthermore, the calculated maximum velocity was 80 m/s (Evans *et al.* 1994), almost identical to the 82 m/s estimated by using a version of Körner's dynamic model (Kaiser and Simmons 1990). The event has not been modeled using DAN3D.

3.2 Jonas Creek, Alberta

The prehistoric Jonas Creek rock avalanche in the Jasper region of the Rocky Mountains occurred in quartzite and involved 2-4 x 10^6 m^3 (Bruce 1978).

Calculated runout distance and deposit distribution correlate well with observations when using the DAN-W with the frictional model with φ_b = 20° or with the Voellmy rheology with f = 0.10-0.15 and ξ = 500 m/s² (see Figure 1). There are no velocity estimates for this event. The calculated runout time was 400 s to 450 s.

3.3 Coal Mine Waste Flow Slides, British Columbia

Hungr *et al.* (2002) reported back-analyses of 43 flow slides in coal mine waste from various mines in the Southern Rocky Mountains region of south-eastern British Columbia. The total runout distance of most of the landslides was

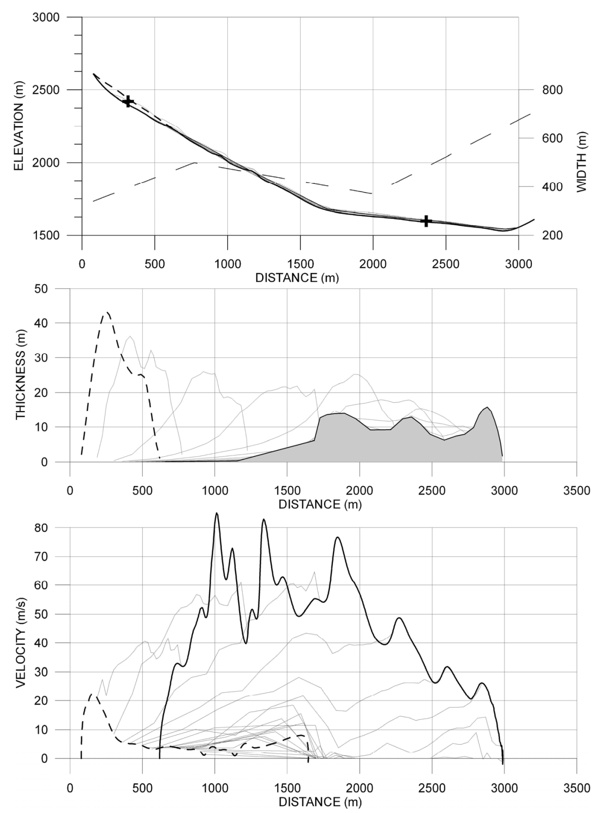

Figure 1. DAN-W back-analysis of the Jonas Creek rock avalanche using the Voellmy rheology with f = 0.15 and ξ = 500 m/s^2.

successfully modeled using the frictional rheology with φ_b = 20-25°. In some cases where the flow entered a confined channel and entrained loose, saturated materials, much greater mobility was observed and it was necessary to use the Voellmy model with f = 0.05-0.1 and ξ = 200 m/s². The relatively low friction coefficient required in these cases suggests that entrainment of liquefied saturated material from the flow path is an important mechanism for increasing the mobility of landslides.

3.4 Frank Slide, Alberta in 1903

A 36 x 10^6 m^3 rock avalanche on Turtle Mountain in the Southern Rockies resulted in approximately seventy fatalities. Fragmented limestone entrained saturated alluvium along the valley floor. At present, a potential smaller landslide from the disturbed South Peak of Turtle Mountain is being intensively studied by the Alberta Geological Survey.

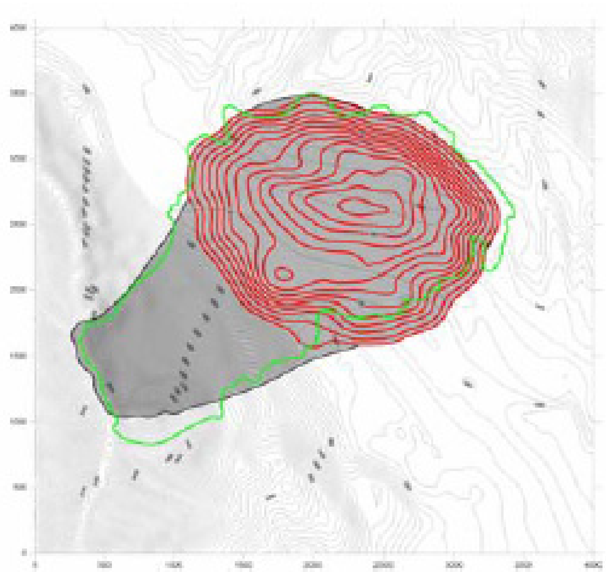

Figure 2. DAN3D back-analysis using Voellmy rheology with f = 0.1 and ξ = 500 m/s².

Back-analyses of the Frank Slide were carried out using both DAN-W and DAN3D. The best overall simulation of the observed runout distance and debris distribution was obtained using the Voellmy rheology with f = 0.1 and ξ = 500 m/s² (see Figure 2). Although exact timing of the event is not known, the calculated time of less than 100 s corresponds with eyewitness reports. The frictional model was also attempted, but the flow duration was significantly shorter (40 s) and the debris distribution was thicker near the Crowsnest River, which contrasts with the observed distal thickening near the highway (McDougall 2006 and unpublished work by authors).

3.5 Hope Slide, British Columbia in 1965

The Hope Slide involved approximately 47.3 x 10^6 m^3 of metamorphic rock in the southern Coast Mountains of B.C. (Mathews and McTaggart 1978). The slide entrained lesser quantities of snow and saturated soils from the valley floor before it buried 4.5 km of B.C. Highway 3. It descended 1160 m to the valley floor and destroyed trees up to a height of several tens of meters on the opposite side of the valley (Cruden 1977).

The slide was analyzed using DAN-W, with the frictional rheology with φ_b = 16° and the Voellmy rheology with f = 0.1 and ξ = 500 m/s² (Hungr and Evans 1996). It was also analyzed in unpublished work by the authors using the DAN3D model where it was found that the frictional rheology with φ_b = 16° or Voellmy rheology with f = 0.15 and ξ = 500 m/s² produced the best results. Both models produced comparable results, although the frictional analysis likely overestimated flow velocities and the Voellmy analysis produced better debris distributions.

3.6 Hummingbird Creek, British Columbia in 1997

The Hummingbird Creek debris flow near Mara Lake in south-central British Columbia started as a debris slide with an initial volume of 25,000 m^3 and entrained debris from a steep, confined creek channel for a total volume of approximately 92,000 m^3. As in many debris flows, the event exhibited longitudinal sorting (Iverson 1997). The flow deposited on a colluvial fan and directly impacted five residential structures and resulted in the flooding of Highway 97A.

The complications of longitudinal variation in rheologies were avoided by using similar rheological properties to those used in the channelized coal waste slope failures, starting with the frictional rheology with φ_b = 20° and transitioning to the Voellmy rheology with f = 0.08 and ξ = 200 m/s² as saturated sediments were entrained (at an average yield rate of approximately 28 m^3/m, based on field observations). The DAN-W analysis was satisfactory in terms of total runout distance and velocity, but less accurate in terms of deposit distribution on the fan. The model accurately calculated the starting point of deposition, but overestimated the end of the runout zone by 100 m with a bimodal depth distribution instead of the tapering profile that was observed in the field (Jakob *et al.* 2000).

3.7 Eagle Pass, British Columbia in 1999

A wedge-shaped block of approximately 75,000 m^3 of metamorphic rock detached from upper valley slopes in Eagle Pass at Clanwilliam Lake near Revelstoke, B.C.. After fragmenting, partly depositing on a bench and then entraining glacial and colluvial material from a steep open slope, the maximum volume in motion was approximately 940,000 m^3. The debris came to rest on the lake ice (Hungr and Evans 2004).

The initial rockslide was modeled in DAN-W as a frictional event, transitioning to the Voellmy rheology as the slide entrained saturated sandy colluvium and till. Good results were produced by selecting parameters within the range established for coal mine waste flows, using the frictional rheology with φ_b = 30° in the proximal path and the Voellmy rheology with f = 0.05 and ξ = 400 m/s² following entrainment (Hungr *et al.* 2002, Hungr and Evans 2004). The deposit distribution between the upper part on the bench and the lower part in the lake were reproduced by the model.

3.8 Nomash River Slide, British Columbia in 1999

A landslide started during the spring snow melt on the upper slope of a glacial valley near Zeballos, on the western coast of Vancouver Island. The initial rock slide involved 300,000 m^3 of marble and basaltic sills. The resulting rock avalanche then entrained an additional 360,000 m^3 of saturated clay, silt, sand, and gravel from the colluvial deposits on the lower slopes (Hungr and Evans 2004)

The DAN-W analysis of this event was similar to the Eagle Pass case, with identical rheological parameters. The transition from frictional to Voellmy rheologies was specified where the entrainment of saturated soils initiated. Again, the calculated debris distribution was matched observations (Hungr and Evans 2004). Corresponding analysis in DAN3D yielded good results, although the calculated flow spread slightly faster than the real event, likely because of the inherent assumption that the rock mass fluidizes instantaneously at failure (McDougall 2006)

3.9 Zymoetz River, British Columbia in 2002

The Zymoetz River rock avalanche began as a 720,000 m^3 slide in volcanic bedrock above a small, steep tributary of the Skeena River in north-western B.C. The slide entrained snow and saturated glacial till and organics to a total volume of 1.4 x 10^6 m^3. The debris flow traveled down the tributary and broke the Pacific Northern Gas pipeline near Terrance, B.C. (Boultbee *et al.* 2006).

The first surge over snow was modeled in DAN3D with the frictional rheology with φ_b = 18°. The main detachment was also simulated using the frictional rheology, with φ_b = 31°, but transitioned to the Voellmy rheology when sufficient saturated material was entrained. Values of f = 0.1 and ξ = 1,000 m/s² in the distal path produced the most satisfactory results in terms of estimated flow velocities and deposit distribution. There is correspondence between observed specific materials that could influence landslide mobility and changes in the calculated flow (McDougall *et al.* 2006, McDougall 2006).

3.10 Pink Mountain, British Columbia in 2002

The Pink Mountain landslide in the Rocky Mountain foothills of the upper Peace River Valley, north-eastern B.C. initiated with 740,000 m^3 of sandstone and shale failing on a relatively gentle open slope. The rock slide grew to 1.04 x 10^6 m^3 by entrainment of clay-rich saturated colluvium from the slope. It buried a forestry road and crossed a creek while covering a vertical distance of 400 m before stopping (Geertsema *et al.* 2005).

The frictional and Voellmy rheological models predict overly-concentrated deposits at the distal end of the deposit area. Using the frictional model with φ_b = 20° at the head and the Bingham model with yield strength τ_{yeild} = 2 kPa and viscosity of $\mu_{Bingham}$ = 5 kPa s below 400 m from the head crown, DAN-W accurately predicted the uniform thickness of the deposited debris sheet, including a thick pocket of blocky debris left behind on the source scar. The calculated velocity distribution – high initial velocity followed by a long period of slow motion – corresponds to an eyewitness account (Geertsema *et al.* 2005). This is the only case of a large landslide back-analysed so far by our group where the optimal fit requires the Bingham plastic rheology. The reason for this is likely related to the relatively high percentage of clay in the material compared to the other case studies.

3.11 McAuley Creek, British Columbia in 2002

The McAuley Creek landslide 30 km east of Vernon in the southern Interior Plateau was part of the same temporal cluster of events as the Zymoetz River landslide. This event initiated as a slide in gneissic rock that had previously been identified as a potential landslide source area. Most of the debris was deposited at the toe of the source slope, damming McAuley Creek, while 1 x 10^6 m^3 of the debris continued down the valley in a 1.6 km long, thin distal deposit. The total slide volume is estimated at 7.4 x 10^6 m^3, primarily from the source with minimal entrainment.

The landslide was modeled in DAN3D using input parameters calibrated through back-analysis of several similar cases (Hungr and Evans 1996). The model used the frictional rheology with φ_b = 30° for the first part of the flow and, at the point the more mobile lobe extended from the main deposit, transitioned to the Voellmy rheology with f = 0.1 and ξ = 500 m/s². The calculated deposit distribution closely matched observations. There are no velocity estimates for this event, but the calculated velocities fall within the reasonable range for events of this magnitude (McDougall 2006).

4. DISCUSSION

Table 1. Summary of the back-calculations. For discussion of landslide classification, please see Hungr *et al.* (2001).

Case	Model	Landslide Classification¹	Dominant Rheology²	Best-fit Parameters φ_b, f, ξ °, unitless, m/s²	References
Avalanche Lake	DAN-W	rock avalanche	Voellmy	N/A, 0.02, 250	Evans *et al.* 1994
Jonas Creek	DAN-W	rock avalanche	frictional	20, N/A, N/A	unpublished work
Jonas Creek	DAN-W	rock avalanche	Voellmy	N/A, 0.1-0.15, 500	unpublished work
Coal Slides	DAN-W	flow slide	frictional	20-25, N/A, N/A	Hungr *et al.* 2002
Coal Slides	DAN-W	flow slide	Voellmy	N/A, 0.05-0.1, 200	Hungr *et al.* 2002
Frank Slide	DAN-W, DAN3D	rock avalanche	Voellmy	N/A, 0.1, 500	unpublished work
Hope Slide	DAN-W	rock avalanche	Voellmy	N/A, 0.1, 500	Hungr and Evans 1996
Hope Slide	DAN3D	rock avalanche	Voellmy	N/A, 0.15, 500	unpublished work
Hummingbird Creek	DAN-W	debris flow	Voellmy	20, 0.08, 200	Jakob *et al.* 2000
Eagle Pass	DAN-W	rock slide - debris avalanche	Voellmy	30, 0.05, 400	Hungr and Evans 2004
Nomash River	DAN-W	rock slide - debris avalanche	Voellmy	30, 0.05, 400	Hungr and Evans 2004
Zymoetz River	DAN3D	debris flow	Voellmy	31, 0.1, 1000	McDougall *et al.* 2006
Pink Mountain	DAN-W	rock slide - debris avalanche	Bingham	20, N/A, N/A τ_{yeild} = 2 kPa $\mu_{Bingham}$ = 5 kPa s	Geertsema *et al.* 2005

¹ For discussion of classifications, please see Hungr and Evans 2004.
² In some cases, frictional rheology was specified on the source scar of the initiating rock slide.

The Canadian cases presented here are consistent with the patterns evident in case sets from other countries. See Table 1 for a summary of the parameters obtained from the back-analyses presented above. All cases were simulated well using the frictional and Voellemy rheologies, except those with high clay content. The Bingham rheology best models the single case studied, providing good runout distance and deposit distribution.

4.1 Rock avalanches

For rock avalanches, the frictional resistance model typically produces reasonable simulations of the observed runout distance. Hungr and Evans (1996) and Pirulli (2005) used DAN-W to back-analyze 34 different rock avalanches using the frictional resistance model. The calibrated φ_b values ranged between 8º and 23º with a mean of 16º. For the Canadian cases, the best result with the frictional rheology falls within this range with and φ_b of 20º.

Hungr and Evans (1996) also noted that the Voellmy rheology produced consistently good debris distribution, velocity profile, and runout distance for f values between 0.03 and 0.24 and ξ between 100 and 1000 m/s^2. Only events involving runout across a glacier or substantial entrainment combined with channelization had calibrated coefficients less than 0.1. For the Canadian cases, the best results were obtained with Voellmy as the dominant rheology, with f between 0.02-0.15 and ξ between 250-500 m/s².

4.2 Flow slides and rock slide–debris avalanches

The frictional rheology can be used for dry friction in the source area, with a transition to the Voellmy model where significant entrainment of saturated soil begins. A similar scheme was used for the coal waste flow slides. A switch from frictional to Voellmy models is often required to produce satisfactory simulation when events initate on open slopes and then become channelized (Ayotte and Hungr 2000, Hungr and Evans 2004).

4.3 Debris flows

For debris flows, the Voellmy model typically produces better simulations, particularly regarding velocities along the path. Based on back-analyses of debris flows worldwide using DAN, calibrated values of f typically range between 0.07 and 0.2, while values of ξ range between 100 and 600 m/s^2 (Hungr *et al.* 1998; Ayotte and Hungr 2000; Jakob *et al.* 2000; Hürlimann *et al.* 2003; Revellino *et al.* 2004; Bertolo and Wieczorek 2005). In the cases presented here, debris flows can be simulated well using the Voellmy rheology, although the parameters are less constrained than avalanches with an f ranging from 0.08 to 0.1 and ξ ranging from 200 to 1,000 m/s².

5. CONCLUSIONS

The systematic collection of back-analyses results should eventually produce a reliable set of guidelines for a priori selection of rheological models and parameters for various types of landslides. Ranges or parameter values provide adequate constraint for forward-prediction modeling in most practical cases. As a result, forward-predictions should be performed as sensitivity analyses over a range of possible parameter values, in order to identify regions of probable landslide runout for use in risk assessment and management.

This brief review of back-analyses allows us to present some generalizations regarding the selection of rheologies and parameters required for a variety of landslide types. This work is presently continuing. Development of the existing model will improve quantitative landslide hazard analysis, an essential contribution to landslide risk management.

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